Higgs Decay into Gravitons: Trees and Loops

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The decay of the Higgs particle into two gravitons was previously calculated by us using a Born term from the Einstein field equations. Subsequently, others computed the same decay via one loop diagrams but omitting the Born terms. Here all of the diagrams up to one loop are discussed, and it is shown that the Born term is overwhelmingly dominant in agreement with our original results.

From the viewpoint of the standard electroweak model, all of the inertial mass is due to the Higgs particles so it is quite evident that almost all of experimental gravity must be the result of the Higgs field; i.e. the inertial and gravitational mass of an object are the same. With this physics in mind we calculated [1] the decay of the Higgs into two gravitons

$$H \to g + g. \tag{1}$$

The action for the decay was computed from the trace of the stress tensor

$$T = g^{\mu\nu} T_{\mu\nu} \tag{2}$$

via the action

$$S_{eff} = \left(\frac{1}{c\,\langle\phi\rangle}\right) \int \chi T d\Omega \tag{3}$$

where the Higgs scalar field is written $\phi = \langle\phi\rangle + \chi$ and the space time volume element $d\Omega = \sqrt{-g}d^4x$. To obtain the Born term, shown below in Fig.1, we employed the Einstein field equations for the stress tensor

$$T_{\mu\nu} = \left(\frac{c^4}{8\pi G}\right)\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right). \tag{4}$$

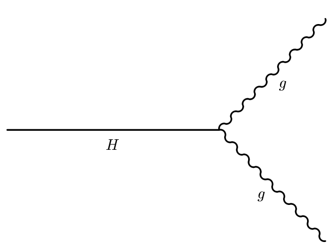

FIG. 1. Shown is the Higgs to two graviton Born term.

Subsequent to our work, the decay in Eq.(1) was computed via the quantum corrections [2] to the stress. A typical one loop diagram which was considered is shown below in Fig.2. The one loop order correction to the stress tensor (the gravitational Casimir effect) has the form [3]

$$T^{(1)}_{\mu\nu} = \left(\frac{\hbar c}{2880\pi^2}\right) \times$$

$$\left(a_1 R^{\alpha\beta} R_{\alpha\mu\beta\nu} + a_2 R^2 g_{\mu\nu} + a_3 \frac{\delta}{\delta g^{\mu\nu}} \int R^2 d\Omega\right), \tag{5}$$

where the coefficients a_1, a_2 and a_3 are dimensionless and of order unity.

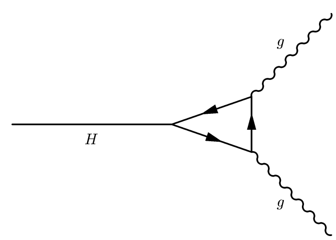

FIG. 2. Shown is a Higgs to two graviton decay contribution which proceeds via an internal Fermion loop.

The full expression for T, including the Einstein (classical tree diagrams) and the Casimir (one loop) correction terms may be written

$$T_{total} = -\left(\frac{c^4 R}{8\pi G}\right) + \left(\frac{\hbar c}{2880\pi^2}\right) \times$$

$$\left(b_1 R^2 + b_2 R^{\mu\nu} R_{\mu\nu} + b_3 R^{\mu\nu\alpha\beta} R_{\mu\nu\alpha\beta} + b_4 \Box R\right) + \ldots\,. \tag{6}$$

where $\{b_j\}$ $(j = 1, 2, 3, 4)$ are (again) constants of order unity.

In our previous work we employed the first term, which contains the Newtonian coupling strength G but not Planck's constant . Our work includes the stress tensor term which contains almost all of usually discussed gravity effects which have been observed, or at least may soon be observed [4]. The stress tensor term we included contains the Newtonian gravitational law, the perihelion anomaly in the orbit of Mercury, gravitational waves and the bending of light around the Sun.

The Einstein term, which contains the Newtonian coupling strength was thrown away [2] by the workers

who computed Eq.(1) employing *only the second term* in Eq.(6). This amounts to treating very precisely the very tiny terms and throwing away the very big terms. For example, if the Higgs-gravity interaction were really of the strength of the gravitational Casimir effect, then the Higgs induced inertial mass would never induce a sufficient gravitational strength to explain Kepler's laws of planetary orbits. The Higgs mechanism would already be ruled out if only one loop gravity terms were to be considered.

To see in detail from where the large terms arise, one may examine the gravitational action

$$S_g = \left(\frac{c^3}{16\pi G}\right)\int R d\Omega, \tag{7}$$

and write R as [5,6]

$$R = \left(\frac{1}{\sqrt{-g}}\right)\partial_\mu\left(\sqrt{-g}W^\mu\right) + \tilde{R}, \tag{8}$$

where $\tilde{R}$ and W^μ depend only on $g_{\mu\nu}$ and $\Gamma^\mu_{\lambda\nu}$, i.e. on the metric and its first derivatives. The decomposition in Eq.(8) depends on the gauge (i.e. coordinate representation) for $g_{\mu\nu}$, but the sum is gauge invariant. For tree diagrams, $\hbar \to 0$ in Eq.(6), one finds from Eqs.(3) and (8)

$$S_{eff} = \left(\frac{c^3}{8\pi G\langle\phi\rangle}\right)\int\left(W^\mu\partial_\mu\chi - \tilde{R}\chi\right)d\Omega. \tag{9}$$

Eq.(9) generates all of the *tree diagrams* for the Higgs to decay into an arbitrary number N gravitons. The tree level coupling depends only on the fields and their first derivatives. This is essential in any field theory. Each emitted graviton carries a factor $\propto \sqrt{G}$ as can be seen in Eq.(13) below.

For small deviations from flat space time,

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \tag{10}$$

and in the gravitational Lorentz gauge, the $H \to g + g$ tree diagram of Fig.1 is generated from Eq.(9) using the second order term

$$R^{(2)} = \left(\frac{1}{4}\right)\left(\partial_\lambda h^{\mu\nu}\partial^\lambda h_{\mu\nu}\right) - \left(\frac{1}{8}\right)\left(\partial_\lambda h\partial^\lambda h\right), \tag{11}$$

where $h = \eta^{\mu\nu}h_{\mu\nu}$. Thus, one finds the action for the tree diagram in Fig.1

$$S(H \to g+g) = -\left(\frac{c^3}{8\pi G\langle\phi\rangle}\right)\int R^{(2)}\chi d^4x. \tag{12}$$

In operator form, one sees that the action in Eqs.(11) and (12) can create or destroy two gravitons by inserting the second quantized field

$$h_{\mu\nu}(\mathbf{r},t) = \sqrt{\frac{32\pi\hbar G}{c^3}}\sum_{\lambda=\pm 2}\int\left(\frac{d^3\mathbf{k}}{(2\pi)^3 2|\mathbf{k}|}\right)\times$$
$$\left\{a(\mathbf{k},\lambda)\epsilon_{\mu\nu}(\mathbf{k},\lambda)e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)} + a^\dagger(\mathbf{k},\lambda)\epsilon^*_{\mu\nu}(\mathbf{k},\lambda)e^{-i(\mathbf{k}\cdot\mathbf{r}-\omega t)}\right\} \tag{13}$$

where $\omega = c|\mathbf{k}|$, the physical helicity polarizations obey $\epsilon_{\mu\nu}(\mathbf{k},\lambda)\epsilon^{\mu\nu *}(\mathbf{k},\lambda') = \delta_{\lambda,\lambda'}$, and

$$\left[a(\mathbf{k},\lambda), a^\dagger(\mathbf{k}',\lambda')\right] = (2\pi)^3(2|\mathbf{k}|)\delta^{(3)}(\mathbf{k}-\mathbf{k}')\delta_{\lambda\lambda'}. \tag{14}$$

It is *very remarkable* that the Newtonian coupling strength cancels out when all of the Eqs.(11), (12) and (13) are taken into account. The tree diagram of Fig.1 yields a transition rate *independent* of G.

The reader may wish to check that the Feynman quadratic action [7]

$$S^{(2)}[h] = \left(\frac{c^3}{16\pi G}\right)\int R^{(2)}d^4x, \tag{15}$$

when employed in the path integral for the stress action $W[T]$,

$$e^{(iW[T]/\hbar)} = \int e^{(iS_g[h]/\hbar)}\, e^{(i\int T^{\mu\nu}h_{\mu\nu}d^4x/2\hbar c)}\,\mathcal{D}h, \tag{16}$$

leads to the Schwinger result

$$W[T] =$$
$$\frac{1}{2c^5}\int d^4x\int d^4y T^{\mu\nu}(x)D_{\mu\nu\lambda\sigma}(x-y)T^{\lambda\sigma}(y). \tag{17}$$

The graviton propagator is given by

$$D_{\mu\nu\lambda\sigma}(x-y) =$$
$$\left(\eta_{\mu\lambda}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\lambda} - \eta_{\mu\nu}\eta_{\lambda\sigma}\right)D(x-y), \tag{18}$$

where Newton's gravitational law is made manifest by

$$D(x) = -\int\left(\frac{4\pi G}{Q^2 - i0^+}\right)e^{iQ\cdot x}\left(\frac{d^4Q}{(2\pi)^4}\right). \tag{19}$$

Schwinger has argued [4] that Eq.(17), which is based on Eq.(11), describes all of experimental Newtonian gravity and most of experimental general relativity. This is as reliably locally Lorentz invariant as is presently possible.

Finally, the tree diagram of Fig.1 leads to the Higgs decay rate

$$\Gamma(H \to g+g) = \left(\frac{\sqrt{2}}{16\pi}\right)\left(\frac{G_F M_H^2}{\hbar c}\right)\left(\frac{M_H c^2}{\hbar}\right), \tag{20}$$

where G_F is the Fermi coupling strength, $(\sqrt{2}\langle\phi\rangle^2) = c^3/(\hbar G_F)$, and M_H is the Higgs particle. We stand by our previously reported result.

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[6] L. D.Landau and E.M. Lifshitz, *Classical Theory of Fields*, pp 268-270, Butterworth-Heinemann Ltd., Oxford (1995).
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